Term-II

CHAPTER 11

THREE DIMENSIONAL GEOMETRY

Syllabus

- ***Direction cosines and direction ratios of a line joining two points. Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Distance of a point from a plane.***

STAND ALONE MCQs (1 Mark each)

Q. 1. Distance of the point (α, β, γ) from y-axis is

(A) β (B) $|\beta|$

(C) $|\beta| + |\gamma|$ (D) $\sqrt{\alpha^2 + \gamma^2}$

Ans. Option (D) is correct.

Explanation :

The foot of perpendicular from point $P(\alpha, \beta, \gamma)$ on y-axis is $Q(0, \beta, 0)$.

$\therefore$ Required distance,

$PQ = \sqrt{(\alpha - 0)^2 + (\beta - \beta)^2 + (\gamma - 0)^2} = \sqrt{\alpha^2 + \gamma^2}$

Q. 2. If the direction cosines of a line are k, k, k, then

(A) $k > 0$ (B) $0 < k < 1$

(C) $k = 1$ (D) $k = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$

Ans. Option (D) is correct.

Explanation :

Since, direction cosines of a line are k, k and k.

$\therefore l = k, m = k$ and $n = k$

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow k^2 + k^2 + k^2 = 1$

$\Rightarrow k^2 = \frac{1}{3}$

$\therefore k = \pm\frac{1}{\sqrt{3}}$

Q. 3. The distance of the plane $\vec{r}.\left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 1$ from the origin is

(A) 1 (B) 7

(C) $\frac{1}{7}$ (D) None of these

Ans. Option (A) is correct.

Explanation:

The distance of the plane $\vec{r}\cdot\left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 1$ from the origin is 1.

[Since $\vec{r}\cdot\hat{n} = d$ is the form of above equation, where d represents the distance of plane from the origin, *i.e.*, $d = 1$]

Q. 4. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

(a) $\frac{10}{6\sqrt{5}}$ (b) $\frac{4}{5\sqrt{2}}$

(c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{\sqrt{2}}{10}$

Ans. Option (D) is correct.

Explanation : We have, the equation of line as

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

This line is parallel to the vector, $\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Equation of plane is $2x - 2y + z = 5$.

Normal to the plane is $\vec{n} = 2\hat{i} - 2\hat{j} + \hat{k}$.

The angle between line and plane is 'θ'.

Then,

$$\sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|}$$

$$= \frac{|(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})|}{\sqrt{3^2 + 4^2 + 5^2}\sqrt{4+4+1}}$$

$$= \frac{|6-8+5|}{\sqrt{50}\sqrt{9}}$$

$$= \frac{3}{15\sqrt{2}}$$

$$= \frac{1}{5\sqrt{2}}$$

$$\sin\theta = \frac{\sqrt{2}}{10}$$

Q. 5. The reflection of the point (α, β, γ) in the xy-plane is

(A) $(\alpha, \beta, 0)$ **(B)** $(0, 0, \gamma)$

(C) $(-\alpha, -\beta, \gamma)$ **(D)** $(\alpha, \beta, -\gamma)$

Ans. Option (D) is correct.

Explanation : In xy-plane, the reflection of the point (α, β, γ) is $(\alpha, \beta, -\gamma)$.

Q. 6. The area of the quadrilateral $ABCD$, where $A(0, 4, 1)$, $B(2, 3, -1)$, $C(4, 5, 0)$ and $D(2, 6, 2)$, is equal to

(A) 9 sq. units **(B)** 18 sq. units

(C) 27 sq. units **(D)** 81 sq. units

Ans. Option (A) is correct.

Explanation :

We have, $A(0, 4, 1)$, $B(2, 3, -1)$, $C(4, 5, 0)$ and $D(2, 6, 2)$

$$\overline{AB} = (2-0)\hat{i} + (3-4)\hat{j} + (-1-1)\hat{k}$$
$$= 2\hat{i} - \hat{j} - 2\hat{k}$$
$$\overline{BC} = (4-2)\hat{i} + (5-3)\hat{j} + (0+1)\hat{k}$$
$$= 2\hat{i} + 2\hat{j} + \hat{k}$$
$$\overline{CD} = (2-4)\hat{i} + (6-5)\hat{j} + (2-0)\hat{k}$$
$$= -2\hat{i} + \hat{j} + 2\hat{k}$$
$$\overline{DA} = (0-2)\hat{i} + (4-6)\hat{j} + (1-2)\hat{k}$$
$$= -2\hat{i} - 2\hat{j} - \hat{k}$$

Thus quadrilateral formed is parallelogram.

∴ Area of quadrilateral $ABCD$

$$= |\overline{AB} \times \overline{BC}|$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= |3\hat{i} - 6\hat{j} + 6\hat{k}|$$

$$= \sqrt{9 + 36 + 36}$$

$$= 9 \text{ sq. units}$$

Q. 7. The locus represented by $xy + yz = 0$ is

(A) A pair of perpendicular lines

(B) A pair of parallel lines

(C) A pair of parallel planes

(D) A pair of perpendicular planes

Ans. Option (D) is correct.

Explanation :

We have,

$$xy + yz = 0$$
$$\Rightarrow \quad x(y+z) = 0$$
$$\Rightarrow \quad x = 0 \text{ and } y + z = 0$$

Above are equations of planes.

Normal to the plane $x = 0$ is $\hat{i}$.

And normal to the plane $y + z = 0$ is $\hat{j} + \hat{k}$.

Now, $\hat{i} \cdot (\hat{j} + \hat{k}) = 0$

So, planes are perpendicular.

Q. 8. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

(A) $\frac{\sqrt{3}}{2}$ **(B)** $\frac{\sqrt{2}}{3}$

(C) $\frac{2}{7}$ **(D)** $\frac{3}{7}$

Ans. Option (C) is correct.

Explanation :

We have equation of plane as $2x - 3y + 6z - 11 = 0$.

Normal to the plane is $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$.

Also x-axis is along the vector $\vec{a} = \hat{i} + 0\hat{j} + 0\hat{k}$.

According to the question,

$$\sin\alpha = \frac{|\vec{a} \cdot \vec{n}|}{|\vec{a}||\vec{n}|}$$

$$= \frac{|\hat{i} \cdot (2\hat{i} - 3\hat{j} + 6\hat{k})|}{\sqrt{1}\sqrt{4+9+36}}$$

$$= \frac{2}{7}$$

Q. 9. Distance between the two planes : $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

(A) 2 units **(B)** 4 units

(C) 8 units **(D)** $\frac{2}{\sqrt{29}}$ unit

Ans. Option (D) is correct.

Explanation :

Distance between two parallel planes,

$Ax + By + Cz = d_1$ and $Ax + By + Cz = d_2$ is

$$\left|\frac{d_1 - d_2}{\sqrt{A^2 + B^2 + C^2}}\right|$$

$$2x + 3y + 4z = 4$$

Comparing with $Ax + By + Cz = d_1$

$A = 2, B = 3, C = 4, d_1 = 4$

And now, $4x + 6y + 8z = 12$

$$2(2x + 3y + 4z) = 12$$

Dividing by 2

$$2x + 3y + 4z = 6$$

Comparing with $Ax + By + Cz = d_2$

$A = 2, B = 3, C = 4, d_2 = 6$

So,

Distance between the two planes

$$= \left|\frac{4-6}{\sqrt{2^2+3^2+4^2}}\right|$$

$$= \left|\frac{-2}{\sqrt{4+9+16}}\right|$$

$$= \frac{2}{\sqrt{29}}$$

Q. 10. The planes : $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

(A) Perpendicular

(B) Parallel

(C) Intersect y-axis

(D) Passes through $\left(0, 0, \frac{5}{4}\right)$

Ans. Option (B) is correct.

Explanation :

Angle between two planes $A_1x + B_1y + C_1z = d_1$ and $A_2x + B_2y + C_2z = d_2$ is given by

$$\cos\theta = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}$$

Given that plane,

$$2x - 1y + 4z = 5$$

Comparing with $A_1x + B_1y + C_1z = d_1$

$A_1 = 2, B_1 = -1, C_1 = 4, d_1 = 5$

$$5x - 2.5y + 10z = 16$$

Multiplying by 2 on both sides,

$$10x - 5y + 20z = 12$$

Comparing with $A_2x + B_2y + C_2z = d_2$

$A_2 = 10, B_2 = -5, C_2 = 20, d_2 = 12$

$$\text{So, } \cos\theta = \left|\frac{(2\times10)+(-1\times-5)+(4\times20)}{\sqrt{2^2+(-1)^2+4^2}\sqrt{10^2+(-5)^2+20^2}}\right|$$

$$= \left|\frac{20+5+80}{\sqrt{4+1+16}\sqrt{100+25+400}}\right|$$

$$= \left|\frac{105}{\sqrt{21}\sqrt{525}}\right|$$

$$= \left|\frac{105}{\sqrt{21}\times\sqrt{25\times21}}\right|$$

$$= \left|\frac{105}{\sqrt{21}\times5\sqrt{21}}\right|$$

$$= \left|\frac{105}{21\times5}\right|$$

$$= 1$$

So, $\cos\theta = 1$

$\therefore \theta = 0°$

Since angle between the planes is 0°.

Therefore, the planes are parallel.

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$ the equation to the sphere whose centre is at (–2, 3, 4) and radius is 6 units.

Reason (R): Given:

Centre is at (–2, 3, 4) and $r = 6$

$\Rightarrow (x_0, y_0, z_0) = (-2, 3, 4)$ and $r = 6$

We know that general equation of sphere is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

$$\Rightarrow (x - (-2))^2 + (y - 3)^2 + (z - 4)^2 = 6^2$$

$$\Rightarrow (x + 2)^2 + (y - 3)^2 + (z - 4)^2 = 6^2$$

$$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9 + z^2 - 8z + 16 = 36$$

$$\Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z + 29 = 36$$

$$\Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A): If two lines are in the same plane *i.e.*, they are coplanar, they will intersect each other if they are non-parallel. Hence the shortest distance between them is zero.

If the lines are parallel then the shortest distance between them will be the perpendicular distance between the lines *i.e.*, the length of the perpendicular drawn from a point on one line onto the other line.

Reason (R): The angle between the lines with direction ratio $\langle a_1, b_1, c_1\rangle$ and $\langle a_2, b_2, c_2\rangle$ is given by:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Ans. Option (B) is correct.

Explanation: Assertion (A) and Reason (R) both are individually correct.

Q. 3. Assertion (A): Direction cosines of a line are the sines of the angles made by the line with the negative directions of the coordinate axes.

Reason (R): The acute angle between the lines $x - 2 = 0$ and $\sqrt{3x - y - 2}$ is 30°.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong.

Since, direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.

Reason (R) is correct.

Since, the slope of the line $x - 2 = 0$ is ∞.

The slope of line $\sqrt{3}x - y - 2 = 0$ is $\sqrt{3}$.

Let $m_1 = \infty$, $m_2 = \sqrt{3}$ and the angle between the given lines is θ.

$$\Rightarrow \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \left|\frac{\frac{m_2}{m_1} - 1}{\frac{1}{m_1} + m_2}\right|$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30°$$

Q. 4. Assertion (A): P is a point on the line segment joining the points (3, 2, –1) and (6, –4, –2). If x coordinate of P is 5, then its y coordinate is –2.

Reason (R): The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ will be perpendicular, iff $aa' + bb' + cc' = 0$.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.

Since $P = (5, y, z)$

Equation of line joining (3, 2, –1) and (6, –4, –2) is

$$\frac{x-3}{6-3} = \frac{y-2}{-4-2} = \frac{z+1}{-2+1} = \frac{x-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

so if point P lies on the line then it must satisfy the above equation

$$\frac{5-3}{3} = \frac{y-2}{-6} = \frac{z+1}{-1}$$

$$\frac{5-3}{3} = \frac{y-2}{-6}$$

Hence y co-ordinate of P is –2.

Reason (R) is false.

Since, the two lines $x = ay + b$, $z = cy + d$ and $= a'y + b'$, $z = c'y + d'$ will be perpendicular, iff $aa' + cc' + 1 = 0$.

Q. 5. Assertion (A): The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$ is 90°

Reason (R): Skew lines are lines in different planes which are parallel and intersecting.

Ans. Option (C) is correct.

Explanation: Assertion (A) is correct.

Given : $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$

and $\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$

Direction ratios of lines are $a_1 = 2, b_1 = 5, c_1 = 4$ and $a_2 - 1, b_2 - 2, c_2 - -3$

As we know, The angle between the lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\left(\sqrt{a_1^2+b_1^2+c_1^2}\right).\left(\sqrt{a_2^2+b_2^2+c_2^2}\right)}$$

$$\Rightarrow \quad \cos\theta = \frac{2\times1 + 5\times2 + 4\times-3}{\left(\sqrt{2^2+5^2+4^2}\right).\left(\sqrt{1^2+2^2+(-3)^3}\right)}$$

$$= 0$$

$$\therefore \quad \theta = 90°$$

Reason (R) is wrong.

In the space, there are lines neither intersecting nor parallel, such pairs of lines are non-coplanar and are called skew lines.

Q. 6. Assertion (A): The length of the intercepts on the co-ordinate axes made by the plane $5x + 2y + z - 13 = 0$ are $\frac{13}{5}, \frac{13}{2}$, 13 unit

Reason (R): Given:

Equation of plane

$$5x + 2y + z - 13 = 0$$

$$\Rightarrow \quad 5x + 2y + z = 13$$

$$\Rightarrow \quad \frac{5x + 2y + z}{13} = 1$$

$$\Rightarrow \quad \frac{x}{\frac{13}{5}} + \frac{y}{\frac{13}{2}} + \frac{z}{\frac{13}{1}} = 1$$

$\therefore$ Length of intercepts are $\frac{13}{5}, \frac{13}{2}$, 13 units

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A cricket match is organized between two Clubs A and B for which a team from each club is chosen. Remaining players of Club A and Club B are respectively sitting on the plane represented by the equation $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})=3$ and $\vec{r}.(\hat{i}+3\hat{j}+2\hat{k})=8$, to cheer the team of their own clubs.

[CBSE QB-2021]

Q. 1. The Cartesian equation of the plane on which players of Club A are seated is

(A) $2x - y + z = 3$ (B) $2x - y + 2z = 3$

(C) $2x - y + z = -3$ (D) $x - t + z = 3$

Ans. Option (A) is correct.

Explanation:

$$\vec{r} = r\hat{i}+y\hat{j}+z\hat{k}$$
$$\vec{r}.(2\hat{i}-\hat{j}+\hat{k}) = 3$$
$$(x\hat{i}+y\hat{i}+z\hat{k})(2\hat{i}-\hat{j}+\hat{k}) = 3$$
$$2x-y+z = 3$$

Q. 2. The magnitude of the normal to the plane on which players of club B are seated, is

(A) $\sqrt{15}$ (B) $\sqrt{14}$

(C) $\sqrt{17}$ (D) $\sqrt{20}$

Ans. Option (B) is correct.

Explanation:

$$\vec{n} = 2\hat{i}+3\hat{j}+2\hat{k}$$
$$|\vec{n}| = \sqrt{1+9+4}$$
$$= \sqrt{14} \text{ units}$$

Q. 3. The intercept form of the equation of the plane on which players of Club B are seated is

(A) $\frac{x}{8}+\frac{y}{8/3}+\frac{z}{8/3}=1$ (B) $\frac{x}{5}+\frac{y}{8/3}+\frac{z}{8/3}=1$

(C) $\frac{x}{8}+\frac{y}{8/3}+\frac{z}{4}=1$ (d) $\frac{x}{8}+\frac{y}{7}+\frac{z}{2}=1$

Ans. Option (C) is correct.

Explanation:

$$\vec{r}.(\hat{i}+3\hat{j}+2\hat{k}) = 8$$
$$(x\hat{i}+y\hat{j}+2\hat{k})(\hat{i}+3\hat{j}+2\hat{k}) = 8$$
$$x+3y+2z = 8$$
$$\frac{x}{8}+\frac{3y}{8}+\frac{2z}{8} = 1$$
$$\frac{x}{8}+\frac{y}{8/3}+\frac{z}{4} = 1$$

Q. 4. Which of the following is a player of Club B?

(A) Player sitting at (1, 2, 1)

(B) Player sitting at (0, 1, 2)

(C) Player sitting at (1, 4, 1)

(D) Player sitting at (1, 1, 2)

Ans. Option (D) is correct.

Q. 5. The distance of the plane, on which players of Club B are seated, from the origin is

(A) $\frac{8}{\sqrt{14}}$ units (B) $\frac{6}{\sqrt{14}}$ units

(C) $\frac{7}{\sqrt{14}}$ units (D) $\frac{9}{\sqrt{14}}$ units

Ans. Option (A) is correct.

Explanation: We know

$$\vec{r}.\hat{n} = d$$
$$\therefore \quad \vec{r}.\frac{(\hat{i}+3\hat{j}+2\hat{k})}{\sqrt{14}} = \frac{8}{\sqrt{14}}$$
$$\therefore \quad d = \frac{8}{\sqrt{14}} \text{ units}$$

II. Read the following text and answer the following questions on the basis of the same:

The Indian coast guard, while patrolling, saw a suspicious boat with people. They were nowhere looking like fishermen. The coast guard were closely observing the movement of the boat for an opportunity to seize the boat. They observed that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of the coast guard helicopter and the boat is (1, 3, 5) and (2, 5, 3) respectively. **[CBSE QB 2021]**

Q. 1. If the line joining the positions of the helicopter and the boat is perpendicular to the plane in which the boat moves, then the equation of the plane is

(A) $-x + 2y - 2z = 6$ (B) $x + 2y + 2z = 6$

(C) $x + 2y - 2z = 6$ (D) $x - 2y - 2z = 6$

Ans. Option (C) is correct.

Q. 2. If the coast guard decide to shoot the boat at that given instant of time, then what is the distance (in meters) that the bullet has to travel?

(A) 5m (B) 3m

(C) 6m (D) 4m

Ans. Option (B) is correct.

Explanation:

$$\vec{n} = \hat{i} + 2\hat{j} - 2\hat{k}$$

Distance that the bullet has to travel

$$= \sqrt{1+4+4}$$
$$= 3\text{m}$$

Q. 3. If the coast guard decides to shoot the boat at that given instant of time, when the speed of bullet is 36 m/sec, then what is the time taken for the bullet to travel and hit the boat?

(A) $\frac{1}{8}$ seconds (B) $\frac{1}{14}$ seconds

(C) $\frac{1}{10}$ seconds (D) $\frac{1}{12}$ seconds

Ans. Option (D) is correct.

Explanation: Time taken for the bullet to travel and hit the boat

$$= \frac{3\text{m}}{36\text{m / sec}}$$
$$= \frac{1}{12} \text{ seconds}$$

Q. 4. At that given instant of time, the equation of line passing through the positions of the helicopter and boat is

(A) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-5}{-2}$

(B) $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-2}$

(C) $\frac{x+1}{-2} = \frac{y-3}{-1} = \frac{z-5}{-2}$

(D) $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+5}{2}$

Ans. Option (A) is correct.

Explanation: Here, direction cosines are 1, 2, –2. Equation of line passing through the positions of the helicopter and boat is

$$\frac{x-x_1}{l} + \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\Rightarrow \quad \frac{x-1}{1} = \frac{y-3}{2} = \frac{z-5}{-2}$$

Q. 5. At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat if the coast guard shoots the bullet along the line whose equation is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{1}$ for the bullet to hit the boat?

(A) $\left(\frac{-8}{3}, \frac{19}{3}, \frac{-14}{3}\right)$ (B) $\left(\frac{8}{3}, \frac{-19}{3}, \frac{-14}{3}\right)$

(C) $\left(\frac{8}{3}, \frac{-19}{3}, \frac{14}{3}\right)$ (D) None of these

Ans. Option (D) is correct.

III. Read the following text and answer the following questions on the basis of the same:

The equation of motion of a missile are $x = 3t$, $y = -4t$, $z = t$, where the time 't' is given in seconds, and the distance is measured in kilometres.

[CBSE QB 2021]

Q. 1. What is the path of the missile?

(A) Straight line (B) Parabola

(C) Circle (D) Ellipse

Ans. Option (A) is correct.

Q. 2. Which of the following points lie on the path of the missile?

(A) (6, 8, 2) (B) (6, –8, –2)

(C) (6, –8, 2) (D) (–6, –8, 2)

Ans. Option (C) is correct.

Explanation: (6, –8, 2) point lie on the path of the missile.

$\because \quad x = 3t, y = -4t, z = t$

at $\quad t = 2$

$x = 6, y = -8, z = 2$

i.e., (6, –8, 2)

Q. 3. At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?

(A) $\sqrt{550}$ kms (B) $\sqrt{650}$ kms

(C) $\sqrt{450}$ kms (D) $\sqrt{750}$ kms

Ans. Option (B) is correct.

Explanation: Here,

$t = 5$ seconds

$x = 3t = 3 \times 5 = 15$

$y = -4t = -4 \times 5 = -20$

$z = t = 5$

$(x, y, z) \equiv (15, -20, 5)$

Distance from starting point (0, 0, 0)

$$= \sqrt{(15-0)^2 + (-20-0)^2 + (5-0)^2}$$

$$= \sqrt{225 + 400 + 25}$$

$$= \sqrt{650} \text{ kms}$$

Q. 4. If the position of rocket at a certain instant of time is (5, –8, 10), then what will be the height of the rocket from the ground? (The ground is considered as the *xy*-plane).

(A) 12 km (B) 11 km

(C) 20 km (D) 10 km

Ans. Option (D) is correct.

Explanation: Height of the rocket from the ground (*i.e.*, *xy*-plane)

= 10 km

Q. 5. At a certain instant of time, if the missile is above the sea level, where the equation of the surface of sea is given by $2x + y + 3z = 1$ and the position of the missile at that instant of time is (1, 1, 2), then the image of the position of the rocket in the sea is

(A) $\left(\frac{-9}{7}, \frac{-1}{7}, \frac{-10}{7}\right)$ (B) $\left(\frac{9}{7}, \frac{-1}{7}, \frac{-10}{7}\right)$

(C) $\left(\frac{-9}{7}, \frac{-1}{7}, \frac{10}{7}\right)$ (D) $\left(\frac{-9}{7}, \frac{-1}{7}, \frac{10}{7}\right)$

Ans. Option (A) is correct.

IV. Read the following text and answer the following questions on the basis of the same:

Suppose the floor of a hotel is made up of mirror polished Salvatore stone. There is a large crystal chandelier attached to the ceiling of the hotel room. Consider the floor of the hotel room as a plane having the equation $x - y + z = 4$ and the crystal chandelier is suspended at the point (1, 0, 1). **[CBSE QB 2021]**

Q. 1. Find the direction ratios of the perpendicular from the point (1, 0, 1) to the plane $x - y + z = 4$.

(A) (–1, –1, 1) (B) (1, –1, –1)

(C) (–1, –1, –1) (D) (1, –1, 1)

Ans. Option (D) is correct.

Q. 2. Find the length of the perpendicular from the point (1, 0, 1) to the plane $x - y + z = 4$.

(A) $\frac{2}{\sqrt{3}}$ units (B) $\frac{4}{\sqrt{3}}$ units

(C) $\frac{6}{\sqrt{3}}$ units (D) $\frac{8}{\sqrt{3}}$ units

Ans. Option (A) is correct.

Q. 3. The equation of the perpendicular from the point (1, 0, 1) to the plane $x - y + z = 4$ is

(A) $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+5}{2}$

(B) $\frac{x-1}{-2} = \frac{y+3}{-1} = \frac{z-5}{2}$

(C) $\frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{1}$

(D) $\frac{x-1}{2} = \frac{y}{-2} = \frac{z-1}{1}$

Ans. Option (C) is correct.

Explanation: Here,

$l = 1, m = -1, n = 1$

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$$

$$\frac{x-1}{1} = \frac{y-0}{-1} = \frac{z-1}{1}$$

$$\Rightarrow \frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{1}$$

Q. 4. The equation of the plane parallel to the plane $x - y + z = 4$, which is at a unit distance from the point (1, 0, 1) is

(A) $x - y + z + (2 - \sqrt{3})$

(B) $x - y + z - (2 + \sqrt{3})$

(C) $x - y + z + (2 + \sqrt{3})$

(D) Both (A) and (C)

Ans. Option (D) is correct.

Q. 5. The direction cosine of the normal to the plane $x - y + z = 4$ is

(A) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

Ans. Option (B) is correct.

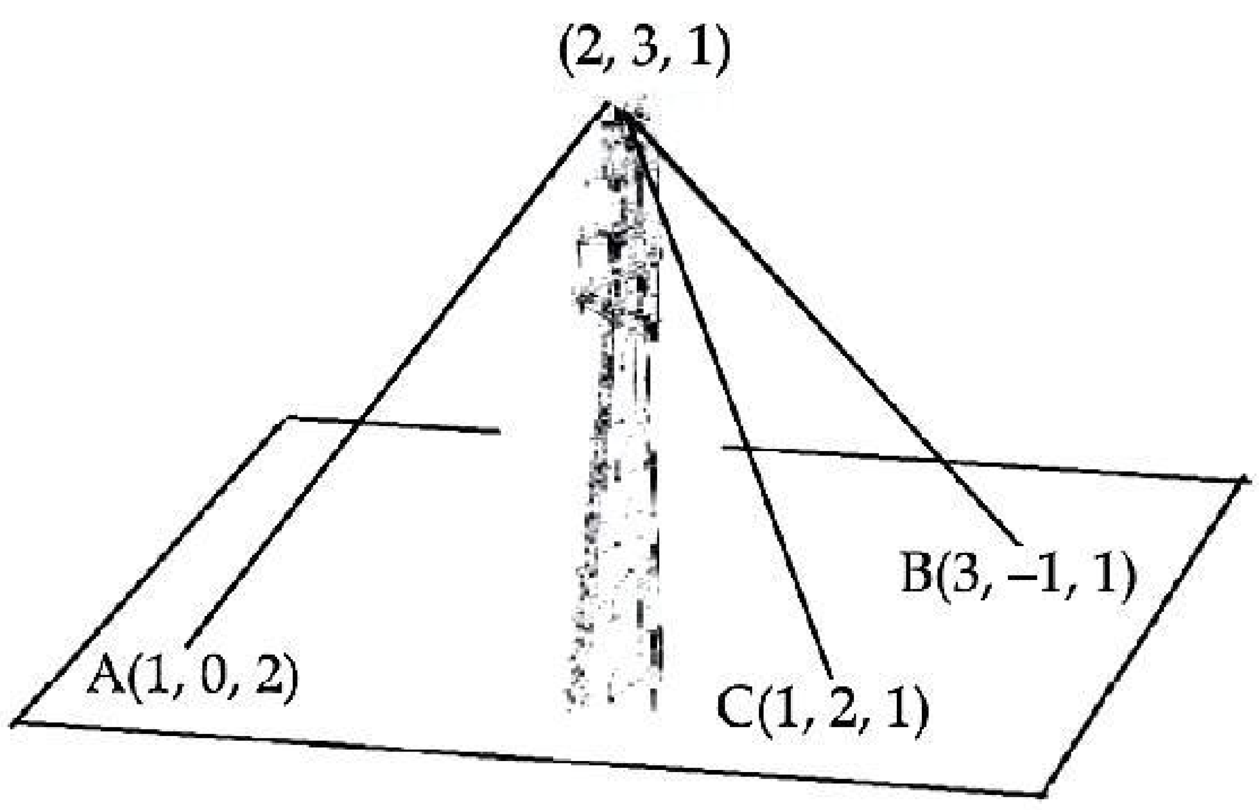
Explanation: Direction cosine of normal to the plane,

$$\left\langle \frac{1}{\sqrt{1^2+(-1)^2+1^2}}, \frac{-1}{\sqrt{1^2+(-1)^2+1^2}}, \frac{1}{\sqrt{1^2+(-1)^2+1^2}} \right\rangle$$

$$\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

V. Read the following text and answer the following questions on the basis of the same:

A mobile tower stands at the top of a hill. Consider the surface on which the tower stands as a plane having points $A(1, 0, 2)$, $B(3, -1, 1)$ and $C(1, 2, 1)$ on it. The mobile tower is tied with 3 cables from the points A, B and C such that it stands vertically on the ground. The top of the tower is at the point (2, 3, 1) as shown in the figure. **[CBSE QB 2021]**



Q. 1. The equation of the plane passing through the points A, B and C is

(A) $3x - 2y + 4z = -11$

(B) $3x + 2y + 4z = 11$

(C) $3x - 2y - 4z = 11$

(d) $-3x + 2y + 4z = -11$

Ans. Option (B) is correct.

Explanation:

$A(1, 0, 2)$, $B(3, -1, 1)$ and $C(1, 2, 1)$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-0 & z-2 \\ 3-1 & -1-0 & 1-2 \\ 1-1 & 2-0 & 1-2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y & z-2 \\ 2 & -1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = 0$$

$$(x-1)(1+2) - y(-2-0) + (z-2)(4-0) = 0$$

$$3x - 3 + 2y + 4z - 8 = 0$$

$$3x + 2y + 4z = 11$$

Q. 2. The height of the tower from the ground is

(A) $\frac{5}{\sqrt{29}}$ units (B) $\frac{7}{\sqrt{29}}$ units

(C) $\frac{6}{\sqrt{29}}$ units (D) $\frac{8}{\sqrt{29}}$ units

Ans. Option (A) is correct.

Q. 3. The equation of the perpendicular line drawn from the top of the tower to the ground is

(A) $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-2}$

(B) $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z-1}{4}$

(C) $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$

(D) $\frac{x+1}{-2} = \frac{y+3}{-1} = \frac{z-5}{2}$

Ans. Option (C) is correct.

Explanation:

$a = 3, b = 2, c = 4$ and

$$(x_1, y_1, z_1) \equiv (2, 3, 1)$$

$$\therefore \quad \frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$$

Q. 4. The co-ordinates of the foot of the perpendicular drawn from the top of the tower to the ground are

(A) $\left(\frac{43}{29}, \frac{-77}{29}, \frac{-9}{29}\right)$ (B) $\left(\frac{9}{7}, \frac{-1}{7}, \frac{-10}{7}\right)$

(C) $\left(\frac{-43}{29}, \frac{77}{29}, \frac{-9}{29}\right)$ (D) $\left(\frac{43}{29}, \frac{77}{29}, \frac{9}{29}\right)$

Ans. Option (D) is correct.

Q. 5. The area of ΔABC is

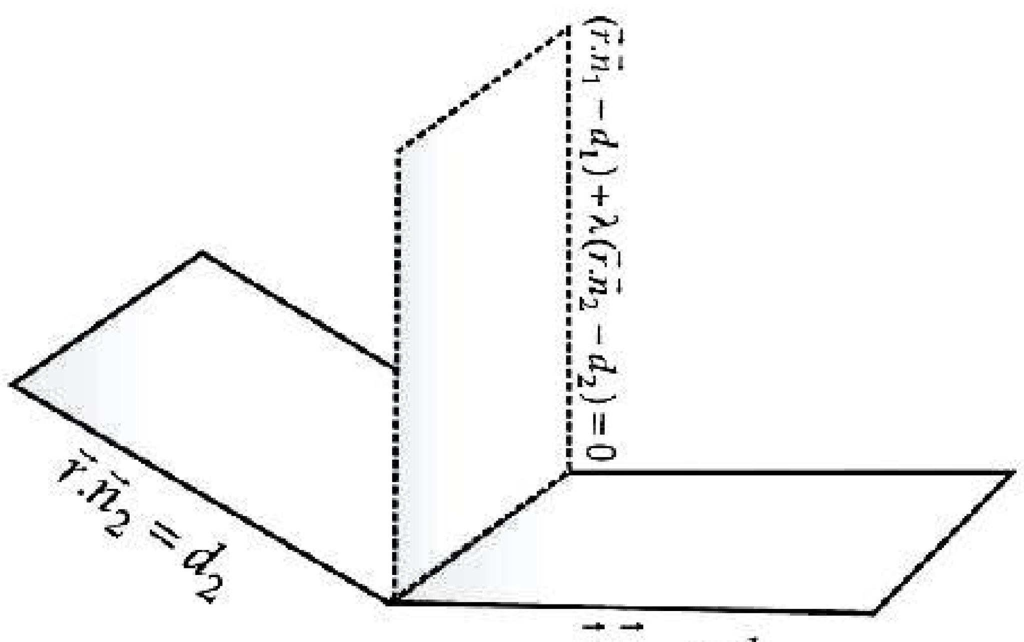
(A) $\frac{\sqrt{29}}{4}$ sq. units (B) $\frac{\sqrt{29}}{2}$ sq. units

(C) $\frac{\sqrt{39}}{2}$ sq. units (D) $\frac{\sqrt{39}}{4}$ sq. units

Ans. Option (B) is correct.

VI. Read the following text and answer the following questions on the basis of the same:

$P_1 : x + 3y - z = 0$ and $P_2 : y + 2z = 0$ are two intersecting planes. P_3 is a plane passing through the point (2, 1, –1) and through the line of intersection of P_1 and P_2.



Q. 1. The angle between P_1 and P_2 is _______.

(A) $\cos^{-1}\left(\frac{1}{5}\right)$ (B) $\cos^{-1}\left(\frac{1}{\sqrt{55}}\right)$

(C) $\cos^{-1}\left(\frac{2}{\sqrt{11}}\right)$ (D) $\cos^{-1}\left(\frac{3}{\sqrt{11}}\right)$

Ans. Option (B) is correct.

Explanation: M

$$\vec{n_1} = \hat{i}+3\hat{j}-\hat{k},\ n_2 = \hat{j}+2\hat{k}$$

$$\cos\theta = \left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right| = \left|\frac{0+3-2}{\sqrt{11}\times\sqrt{5}}\right| = \frac{1}{\sqrt{55}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{55}}\right)$$

Q. 2. Equation of P_3 is _______.

(A) $4x + y - 2z = 10$ (B) $x + y - 2z = 3$

(C) $x + 9y + 11z = 0$ (D) $4x - y + z = 0$

Ans. Option (C) is correct.

Explanation:

Let the equation of P_3 be $P_1 + \lambda P_2 = 0$

i.e., $(x + 3y - z) + \lambda(y + 2z) = 0$

P_3 passes through the point $(2, 1, -1)$

$\therefore$ $(2 + 3 + 1) + \lambda(1 - 2) = 0$

$\Rightarrow$ $\lambda = 6$

$\therefore$ The equation of P_3 is $x + 9y + 11z = 0$

Q. 3. Equation of plane parallel to P_3 and passing through (1, 2, 3) is _______.

(A) $x + 9y + 11z - 52 = 0$

(B) $x + 9y + 11z - 20 = 0$

(C) $4x + y - 2z + 10 = 0$

(D) $4x + y - 2z + 1 = 0$

Ans. Option (A) is correct.

Explanation: The required equation is

$(x - 1) + 9(y - 2) + 11(z - 3) = 0$

i.e., $x + 9y + 11z - 52 = 0$

Q. 4. _______ is a point on P_3.

(A) (1, 2, 3) (B) (–1, 4, 3)

(C) (–6, –3, 3) (D) (6, 3, –3)

Ans. Option (C) is correct.

Explanation: The equation of P_3 is $x + 9y + 11z = 0$.

$\therefore$ (–6, –3, 3) is a point on P_3.

Q. 5. Distance of P_3 from origin is _______ units.

(A) 0 (B) 1

(C) $\frac{1}{\sqrt{5}}$ (D) $\frac{11}{10}$

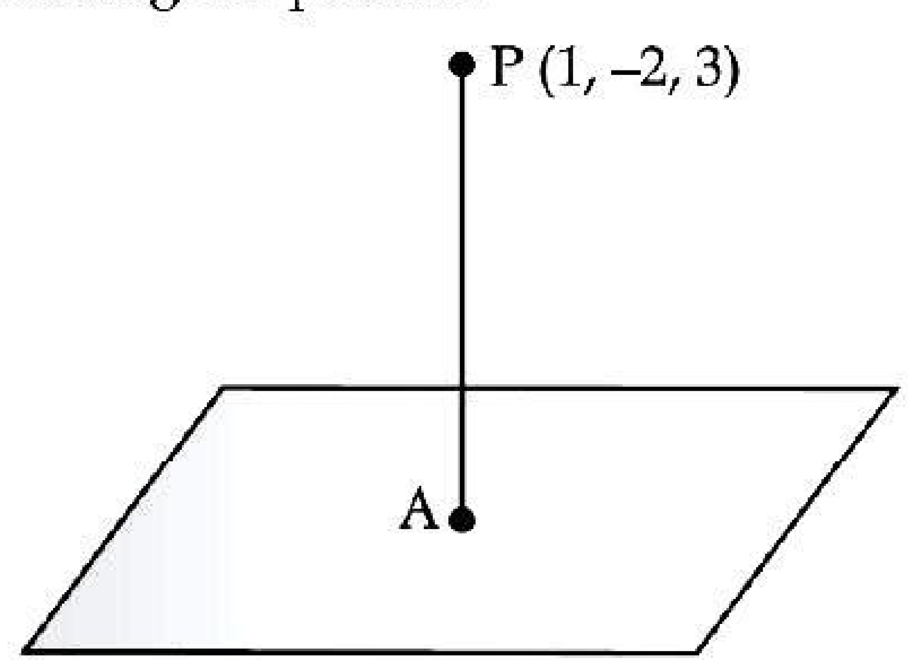
Ans. Option (A) is correct.

Explanation: (0, 0, 0) is a point on P_3.

$\therefore$ Distance of P_3 from origin = 0

VII. Read the following text and answer the following questions on the basis of the same:

Consider the plane $\pi_1 : 2x - 3y + 4z + 9 = 0$ and the point $P(1, -2, 3)$. π_1 is a plane parallel to π_1 and containing the point P.



Q. 1. Equation of π_2 is _______.

(A) $2x - 3y + 4z + 9 = 0$

(B) $2x - 3y + 4z + 20 = 0$

(C) $2x - 3y + 4z - 20 = 0$

(D) $2x - 3y + 4z - 9 = 0$

Ans. Option (C) is correct.

Explanation: Equation of π_2 is

$2(x - 1) - 3(y + 2) + 4(z - 3) = 0$

i.e., $2x - 3y + 4z - 20 = 0$

Q. 2. Distance between π_1 and π_2 is _______ units.

(A) 5 (B) $\sqrt{29}$

(C) $\sqrt{13}$ (D) $2\sqrt{3}$

Ans. Option (B) is correct.

Explanation: Distance between π_1 and π_2

$$= \frac{|-20-9|}{\sqrt{4+9+16}}$$

$$= \sqrt{29} \text{ units}$$

Q. 3. A is the foot of perpendicular from P to π_1. Equation of PA is _______.

(A) $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4}$

(B) $\frac{x+1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$

(C) $\vec{r} = (\hat{i}+2\hat{j}-3\hat{k}) + \lambda(2\hat{i}-3\hat{j}+4\hat{k})$

(D) $\vec{r} = (\hat{i}-2\hat{j}+3\hat{k}) + \lambda(\hat{i}-2\hat{j}+3\hat{k})$

Ans. Option (A) is correct.

Explanation:

Here $\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$

$\vec{b} = \vec{n} = 2\hat{i}-3\hat{j}+4\hat{k}$

Equation of PA is

$$\vec{r} = (\hat{i}-2\hat{j}+3\hat{k}) + \lambda(2\hat{i}-3\hat{j}+4\hat{k})$$

or $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4} = \lambda$

Q. 4. The co-ordinates of A are _______.

(A) (0, 0, 0) (B) (–1, 2, 3)

(C) (–1, 1, –1) (D) (1, 4, –2)

Ans. Option (C) is correct.

Explanation:

Generally $A = (2\lambda + 1, -3\lambda - 2, 4\lambda + 3)$.

A is a point on π_1.

$\Rightarrow 2(2\lambda+1) - 3(-3\lambda-2) + 4(4\lambda+3) + 9 = 0$

$\Rightarrow 29\lambda + 29 = 0$

$\Rightarrow \lambda = -1$

$\therefore A = (-1, 1, -1)$

Q. 5. The image of P on π_1 is _______.

(A) (–1, 1, –1) (B) (–1, 2, –3)

(C) (–3, 4, –5) (D) (0, 0, 0)

Ans. Option (C) is correct.

Explanation:

Let $P'(\alpha, \beta, \gamma)$ be the image A is mid-point of PP'.

i.e., $(-1, 1, -1) = \left(\frac{\alpha+1}{2}, \frac{\beta-2}{2}, \frac{\gamma+3}{2}\right)$

$\Rightarrow \alpha = -3, \ \beta = 4, \ \gamma = -5$

$\therefore P' = (-3, 4, -5)$